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Deposition-rate effects on rough surfaces formed by sedimenting particles

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The quasi-two-dimensional sedimentation of silica particles in a viscous fluid results in quasi-one-dimensional surfaces. These surfaces are rough on all length scales between the particle size and the cell size, but different roughness exponents are observed in two well-defined length scale regimes. Hydrodynamic forces should play an important role in determining which, if either, length scale regime shows universal properties. The role of these hydrodynamic forces can be controlled through control of the deposition rate of particles into the cell. A range of different deposition rates has been examined, and a clear upward trend was observed in the scaling exponent found at long length scales, while the scaling exponent found at short length scales remained relatively constant and very consistent with results of previous experiments with a fixed number of particles but wherein cell length, cell width, and fluid viscosity were all varied with no effect on observed interfacial roughness. [S1063-651X(97)09611-6]

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I. INTRODUCTION

The growth of rough surfaces is a problem of practical importance in many areas. Rough surfaces formed via sedimentation are of particular interest, and their formation involves fundamental nonequilibrium statistical physics [1]. Theoretical efforts [2–34] have emphasized the simple Kardar-Parisi-Zhang (KPZ) surface growth equation with various kinds of added noise [19,23,28,33,34] to accommodate the possible variety of forces which might complicate the dynamics of growth in any one physical system. A further step has been taken by Mehta, Luck, and Needs [35], who take a more local approach to sandpile dynamics, including a nonlinear coupling between moving grains and relatively immobile clusters in sandpiles.

However, the hydrodynamic interactions between sedimenting particles are very complicated and should involve long-range forces caused by the motion of neighboring particles out to a distance set by viscosity and the density of the particles in the settling solution [36,37]. Unfortunately, it is difficult to comment on the exact nature of these long-range forces when we have so many particles settling simultaneously. Analytic sedimentation theory has succeeded only in analyzing the attraction between two settling particles [38], the effective behavior of settling particles in the dilute regime [36,39,40], and some features of many-body interactions between the particles [41,42]. Computer simulations have proven to be quite successful in accounting for interactions among hundreds (but not yet tens of thousands) of particles [43]. In addition, phenomenological analytic theory has described some interesting length scales which appear in the breakup of a line or a plane of settling homogeneous particles [44,45] or of arrays of mixed particles [46]. Beyond this some recent theoretical work [47,48] has heightened pessimism by highlighting how complicated the situation is, interesting experimental results have become available and other recent theoretical work [37,49] holds out some hope of determining the particles' interactions and profiles through wide ranges of volume fraction and Péclet number in sedimentation problems.

The presence of these long-range forces differentiates the study of rough surfaces formed by sedimentation from that of other, apparently similar, rough surfaces. In previous work [50,51], with closed cells and a fixed total number of sedimenting particles, the surfaces formed were shown to be rough on all length scales between the particle size and the cell size. However, different roughness exponents were observed in two distinct length scale regimes, with a well-defined crossover length scale. Both previous exponents were robust against changes in cell length, cell width, and fluid viscosity. A strong similarity at longer length scales between height-height correlations at the rough surface and density-density correlations inside the fluid far above the surface suggested that the roughness at longer length scales is closely tied to the hydrodynamic interactions among particles in the fluid. These hydrodynamic interactions should depend on fluid viscosity and the details of the interparticle distances. In this paper we report experiments in which the deposition rate of the particles into the cell was varied, allowing us to probe the role of these hydrodynamic interactions in the roughness of quasi-one-dimensional interfaces formed by quasi-two-dimensional sedimentation.

II. EXPERIMENTAL METHOD

In the previous work, all measurements were performed with closed cells. The walls of these cells were of $\frac{1}{4}$-in. float glass, held 1 mm apart by sealed side frames of precision-machined Plexiglas. A very large number (40 000) of 0.06-cm-diam monodisperse silica spheres [52] were placed in the cell before the cell was filled with a viscous fluid (such as glycerin) and closed. Each cell could be rotated about a horizontal axis perpendicular to the gap direction. When the cell was rotated, the particles that had been at rest at the bottom

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fell through the glycerin, slowly building a new surface at the bottom of the cell. During and at the end of each such process we photographed the cell (or parts of the cell). All of our photographs were taken with a 35-mm single-lens reflex slide camera. The slides were then digitized to a maximum resolution of 2048×1366 pixels by a Nikon LS 3500 35-mm film scanner. Individual particles were typically resolvable and thus the position of the particles in the interface could be traced very accurately using the image analysis program OPTIMAS. In these experiments, the particles did not follow a ballistic trajectory. There were obvious backflows among the particles, visible to the eye. There was a strong correlation between the height-height correlations measured at the surface at large length scales and the density-density correlations of the particles measured in the flow as the particles fell, making it seem plausible that hydrodynamic forces were involved in setting the roughness at large length scales.

The present experiments were designed to vary the hydrodynamic interactions by controlling the deposition rate of the particles, with a low deposition rate corresponding to weaker hydrodynamic interactions since the particles would on the average be further apart as they fell. To accomplish this, a cell was developed to have dimensions comparable to those of the previous cell, but with an open top where the particles could be added through a funnel, which steadily dropped the particles as it traveled back and forth across the top of the cell. The speed of the funnel sweeping across the cell, along with the size of the funnel and the viscosity of the fluid through which the particles were allowed to fall, allowed us to control the deposition rate over a range of values from 0.7 to 50 particles/sec. This new method of delivery also allowed us to deposit the particles more uniformly in time. In our earlier closed-cell experiment, the flow of the particles to the surface began slowly, built up to a fairly steady rate, and then tapered off with an average deposition rate far in excess of the highest rate studied in the present experiment. In the present case, the delivery rate of the particles to the interface was much more uniform.

III. DISCUSSION

As in the previous work, we define the rms thickness of the interface to be

$$W(L,t)=\left[\frac{1}{N}\sum_{i=1}^{N}\tilde{h}(x_i,t)^2\right]^{1/2}, \tag{1}$$

where $h(x_i,t)$ is the height of the growing interface at horizontal position x_i and time t, $\overline{h(t)}$ is the horizontally averaged interface height at time t,

$$\overline{h(t)}=\frac{1}{N}\sum_{i=1}^{N} h(x_i,t), \tag{2}$$

and

$$\tilde{h}(x_i,t)=h(x_i,t)-\overline{h(t)}. \tag{3}$$

As discussed in Ref. [51] it is not at all clear that our system is in a scaling regime, nor is it obvious that scaling ideas should apply to sedimentation, but a useful way of analyzing our data is to adopt and extend the standard roughness analysis by tentatively accepting a scaling ansatz for rough interface growth [4]. If we follow this ansatz, we write

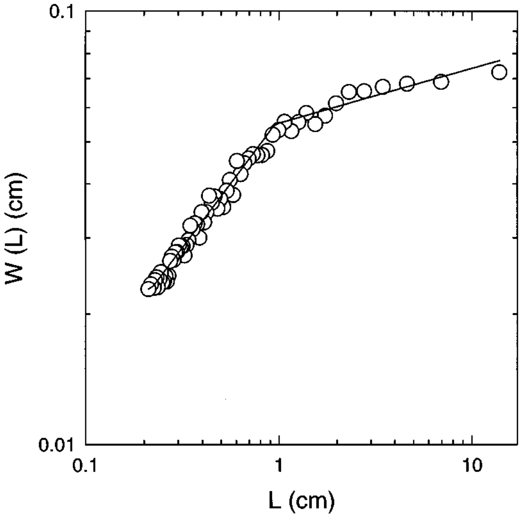

FIG. 1. A typical roughness function $W(L,t)$ for the new, open cell, at a deposition rate of about 0.7 particle/sec.

$$W(L,t)=L^{\alpha}f(t/L^{\alpha/\beta}), \tag{4}$$

where the exponents α and β are the static and dynamic scaling exponents. The function $f(t/L^{\alpha/\beta})$ is expected to have an asymptotic form such that

$$W(L,t)\sim t^{\beta} \quad \text{for } t\ll L^{\alpha/\beta}$$

and

$$W(L,t)\sim L^{\alpha} \quad \text{for } t\gg L^{\alpha/\beta}. \tag{5}$$

Figure 1 shows a typical example of $W(L,t)$ for the new cell. As in the earlier work, to minimize wall effects we have used only the middle 70% of each interface for our analysis. At every deposition rate studied, there appear to be two roughness exponents corresponding to two different length scales, with a crossover length of somewhat less than 1 cm (roughly the same as the previously observed crossover length but slightly smaller). At very low deposition rates (0.7 particle/sec) the exponent α corresponding to length scales larger than 1 cm is approximately 0.2, while that for smaller length scales is approximately 0.6. As the deposition rate is increased, the value of α found at large length scales shows a significant increase, while the value of α found at small length scales remained approximately constant. This behavior is shown in Fig. 2. Over the entire range of deposition rates shown in Fig. 2, the particles fell evenly, and there were no indications of the kind of backflow patterns seen in the flow in the previous work. This strengthens the earlier indications that the large-length-scale exponent is strongly affected by the hydrodynamic interactions whereas the small-length-scale exponent is fairly robust in the presence of significant change of the hydrodynamic interactions in the fluid.

Up to fairly high deposition rates (approximately 30–40 particles/sec), there was no evidence of the type of large undulatory structures seen in the previously reported rough interfaces. In the experiments with closed cells, the surface typically showed one or two hills in the middle, with a mini-

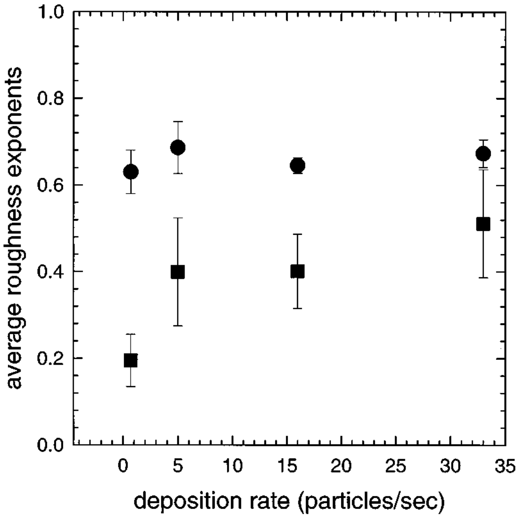

FIG. 2. Average roughness exponents found at various deposition rates. The circles denote α_L, the squares represent α_S. Values of each α are typically averaged over four experimental runs, each of which would have had relatively small uncertainty in α. Thus the stated uncertainties arise from the reproducibility of the roughness.

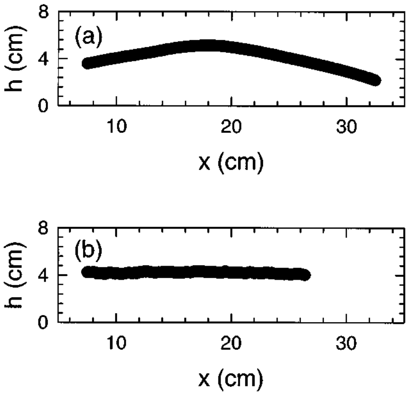

FIG. 3. Interface height as a function of position x averaged over runs. As discussed in the text, 6 cm have been eliminated from each side. (a) The height vs the position for closed cells, averaged over 50 runs, (b) the height vs the position for the new open cell, averaged over four runs.

mum of particles near the wall. When several typical runs were averaged, we obtained the structure shown in Fig. 3(a). Averaged data [shown in Fig. 3(b)] for the present experiment do not exhibit such long-length-scale structures. The low-deposition-rate elimination of such structure correlates with the lack of observable backflow at these low deposition rates. The new design of the open cell, in dramatically decreasing the deposition rate, has thus been successful in significantly decreasing the backflow and demonstrating a concurrent elimination of long-length-scale structures.

Ideally, one could hope to increase the particle deposition rate beyond 40 particles/sec either until the interfacial patterns matched those seen in the closed-cell experiment or until the deposition rates exceeded those of the closed-cell experiments. However, as the deposition rate was increased beyond 40 particles/sec we encountered a new phenomenon. In this regime, the particles formed spatial correlations as they hesitated to overcome the surface tension at the top of the cell and settled with such spatial correlations in evidence. Thus there was an abrupt dynamical transition to flows more like those seen in the closed-cell experiments. At these relatively low deposition rates of 40–70 particles/sec, however, backflows in the fluid appeared at apparently random positions rather than at the cell-size-related positions seen at the very high deposition rates of our closed-cell experiments.

We summarize our results as follows: we have added a set of tests to the previous controls we had placed on growth of rough interfaces by sedimentation in a viscous fluid. In the previous work, at high particle deposition rate, two regimes of robustly reproducible characteristic roughness appeared despite changes in cell length, cell width, and fluid viscosity, with clear correlations between visible backflow patterns in the fluid and long-length-scale roughness of the developing interface. In the present work the deposition rate is varied over a wide range of values, all of which are small in comparison to those typical of the closed-cell work. At the lower end of these new deposition rates, the interface roughness at small length scales is again the same as previously seen, and above an ~1-cm crossover length (consistent with all previous results), the interface is very smooth and distinctly different from the closed-cell results. At the higher, but still low, deposition rates of the present experiment, apparently random particle correlations become visible in the fluid and at the long length scales of the growing interface. The abrupt transition in this long-length-scale roughness appears to depend on the dynamics of breaking surface tension as particles are added to the cell and thus not to indicate universal significance to its onset deposition rate, but its appearance does strengthen the other suggestions that the roughness observed in sedimentation at long length scales depends sensitively on details of the hydrodynamic interactions while the small-length-scale regimes of roughness may arise from universal considerations.

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